Standard Notation and Glossary of Terms

Primitive terms: players, strategies, payoffs/utilities

PLAYERS

Individual players	$i = 1,2,\ldots, n$
The set of all players	$I=\{1,2, \ldots , n\}$
Player's i opponents = all the other players except player i	$-i$

STRATEGIES

Specific strategy of player i	s_i
If we need to enumerate all k strategies of i we write	$s_i^1, s_i^2, \ldots , s_i^k$
The set of all strategies (strategy set) of player i	S_i
Strategy profile is a vector	$s = (s_1, s_2, \ldots , s_n)$
Strategy profile of i's opponents	$s_{-i} = (s_1, s_2, \ldots s_{i-1}, s_{i+1}, \ldots , s_n)$
The set of all strategy profiles	$S = S_1 \times S_2 \times \ldots \times S_n$

PAYOFFS

Payoff function of player i is a function (Re denotes the set of real numbers)	$u_i: S \Rightarrow Re$

Game, G, is defined as a triple consisting of the set of players, the set of all strategy profiles and the set of players' utility functions: $G = \langle I, S, (u_1, u_2, \ldots , u_n) \rangle$

Games in normal form

p ⟶ q means p > q . Think about the arrow as: you would go from p to q.
p ⟷ q means p = q

The dominance relations, an example

	X	Y	Z
A	3 1 ↓	2 1 ↓	2 0 ↓
B	2 3	3 2	0 1
C	1 2	0 1	1 1

Strategy B of the Row player **strictly dominates** his strategy A: for each strategy of the Column player, strategy B gives Row a higher payoff.

	X	Y	Z
A	3 1	2 1	2 0
B	2 3	3 2	0 1
C	1 2 ↑	0 1 ↑	1 1 ↕

Strategy B of the Row player **weakly dominates** his strategy C: for each strategy of the Column player, strategy B gives Row a payoff which is at least as large as the payoff to C and in some cases the payoff is strictly larger.

Using the general notation to define the two concepts of dominance.

	s_2^1		s_2^p		s_2^m
s_1^1	$u_2(s_2^1,s_1^1)$ $u_1(s_1^1,s_2^1)$	.....	$u_2(s_2^p,s_1^1)$ $u_1(s_1^1,s_2^p)$	.....	$u_2(s_2^m,s_1^1)$ $u_1(s_1^1,s_2^m)$
.....	.....	.....	.....	.....	.....
s_1^t	$u_2(s_2^1,s_1^t)$ $u_1(s_1^t,s_2^1)$		$u_2(s_2^p,s_1^t)$ $u_1(s_1^t,s_2^p)$		$u_2(s_2^m,s_1^t)$ $u_1(s_1^t,s_2^m)$
.....	.....	.....	.....	.....	.....
s_1^r	$u_2(s_2^1,s_1^r)$ $u_1(s_1^r,s_2^1)$	.....	$u_2(s_2^p,s_1^r)$ $u_1(s_1^r,s_2^p)$	.....	$u_2(s_2^m,s_1^r)$ $u_1(s_1^r,s_2^m)$
.....	.....	.....	.....	.....	.....
s_1^n	$u_2(s_2^1,s_1^n)$ $u_1(s_1^n,s_2^1)$	.....	$u_2(s_2^p,s_1^n)$ $u_1(s_1^n,s_2^p)$	.....	$u_2(s_2^m,s_1^n)$ $u_1(s_1^n,s_2^m)$

In the game above:

s_1^r *strictly dominates* s_1^t if for all p=1,2,...,n, $u_1(s_1^t,s_2^p) < u_1(s_1^r,s_2^p)$.

s_1^r *weakly dominates* s_1^t if for all p=1,2,...,n, $u_1(s_1^t,s_2^p) \leq u_1(s_1^r,s_2^p)$ and for at least one p, $u_1(s_1^t,s_2^p) < u_1(s_1^r,s_2^p)$.

THE (GENERAL) DEFINITIONS:

In any game $G = \langle I, S, (u_1, u_2, \ldots, u_n) \rangle$, strategy s_i *strictly dominates* strategy s_i^* if for all strategies s_{-i} of i's opponents $u_i(s_i^*,s_{-i}) < u_1(s_i,s_{-i})$; strategy s_i *weakly dominates* strategy s_i^* if for all strategies s_{-i} of i's opponents $u_i(s_i^*,s_{-i}) \leq u_1(s_i,s_{-i})$ and for at least one s_{-i}, $u_i(s_i^*,s_{-i}) < u_1(s_i,s_{-i})$.

We say that strategy s_i *dominates* strategy s_i^* if s_i dominates s_i^* either strictly or weakly; such s_i^* will be called *dominated*.

We say that strategy s_i is *dominant* if it dominates all the other strategies of i.

Strategy s_i is the *best response* to s_{-i} if $u_1(s_i,s_{-i}) \geq u_i(s_i^*,s_{-i})$ for all strategies s_i^* of player I. Denote: $s_i = BR_i(s_{-i})$

A profile of strategies $(s_1, s_2, \ldots, s_n)$ is in Nash equilibrium if for every i=1,...,n, $s_i = BR_i(s_{-i})$.