# Elementary Probability Theory –Definitions

## Simple Experiment:

*A simple experiment is some well-defined act or process that leads to a single well-defined outcome.*

## Events:

The basic elements of probability theory are the possible distinct outcomes of some idealized simple experiment. The set of all possible distinct outcomes for a simple experiment will be called the *sample space* for that experiment.

Any member of the sample space is called a *sample point*, or an *elementary event*.

### Events as Sets of Possibilities

The symbols S or U are often used to represent the *sample space,* or the set of all elementary events for a simple experiment. Capital letters such as *A*, *B*, and so on represent events, each of which is a subset or subgrouping of elementary events in *S*.

$$\text{sample space} = S \text{ or } U$$

Suppose we have two events, *A* and *B*, each composed of elementary events in *S*. Then we can consider an experimental outcome that is a member of both A and *B* –event *A* and *B* has occurred. This is symbolized as:

$$(A \text{ and } B) = (A \cap B)$$

Similarly, we can imagine an outcome that is *either* event A or B. This is known as the *union* of *A* and *B*, given as:

$$(A \text{ or } B) = (A \cup B)$$

By definition $(A \cap B)$ is also $(A \cup B)$, but, the reverse is not true.

Suppose that there is a simple experiment with sample space *S* and that *A* is some event in that sample space. Not only do we have a class of events *A*, but also *not* A given as $\bar{A}$. This is called the *complement*.

In probability theory, any event that cannot be predicted with certainty, when measured, is called a *random variable*. Do not confuse this with a random event. For random variables:

1. Each experiment or event has an outcome that cannot be determined before the event occurs.

2. However, each event or experiment must allow for the description of each and every possible outcome.

The collection of all outcomes is called the outcome or sample space or universe.
Some examples of sample spaces are:

1. A rat from a cage is selected at random and its sex determined. The possible outcomes are male or female; the sample space is $S$ = {M,F}.

2. Each of 6 students selects an integer between 1 and 52. We are interested in whether any two of the selected numbers match or are different; $S$ = {M, D}.

3. A box of cereal contains 1 of 4 different prizes. Buying one box yields one prize as an outcome; $S = \{P_1, P_2, P_3, P_4\}$.

4. The state of Maryland institutes a daily lottery where a three-digit number is randomly selected as the winning number; $S$ = {000, 001, 001, . . . , 999}.

5. A fair coin is flipped until a head is observed. If $K$ is equal to the number of trials necessary to produce a head, then $K$ could be *any* positive integer; $S = \{1, 2, 3, . . . \infty\}$

6. A biologist identifies a marsh bird. An adult bird is captured and weighed. $W$ = the weight of the bird. Based on past experience $S = \{200 \leq W \leq 450\}$.

7. Same biologist, same bird. Now however, the biologist determines the weight *and* sex of the bird, a two-dimensional sample space; $U$ = {(S,W); S=M or F, $200 \leq W \leq 450$}.

Examples 1 through 4 show a finite number of outcomes.
Example 5, the number of outcomes is infinite but countable.
Example 6 shows outcomes represented by a range of possible outcomes.
Example 7 shows a two-dimensional sample space.

# Elementary Probability Theory –Probabilities

Given the sample space $S$, and the family of events in $S$, a probability function associates with each $A$ a real number p($A$), the probability of event $A$, such that the following are true:

1. p($A$) $\geq$ 0 for every event $A$

2. p($S$) = 1.00

If there is a set of countable events $\{A_1, A_2, \ldots, A_n\}$, and if the events are *mutually exclusive*, then:

$$p(A_1 \cup A_2 \cup \ldots \cup A_n) = p(A_1) + p(A_2) + \ldots p(A_n)$$

The probability of the union of mutually exclusive events is the sum of their individual probabilities.

# Elementary Probability Theory –Some Simple Rules

Probability Rule 1: $$p(\overline{A}) = 1 - p(A)$$

Given: $$p(A \cup \overline{A}) = p(A) + \,$$

And: $$p(A \cup \overline{A}) = S$$

Then: $$p(A) + p(\overline{A}) = 1$$

Therefore: $$p(\overline{A}) = 1 - p(A)$$

This is sometimes called the *rule of complementary probability*

Probability Rule 2: $$0 \leq p(A) \leq 1$$

This is the *probability range*.

Probability Rule 3: $$p(o) = 0, \textit{for any } S$$

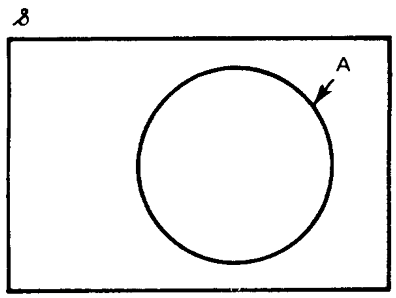
This is the *rule of the impossible event.* That is, the impossible event always has a probability of zero.

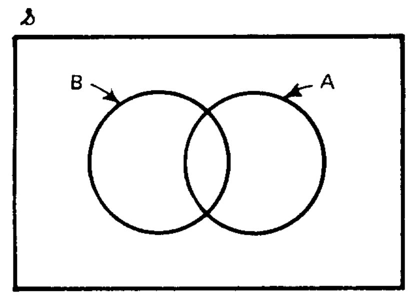
Probability Rule 4. For any two events A and B in S: $$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$

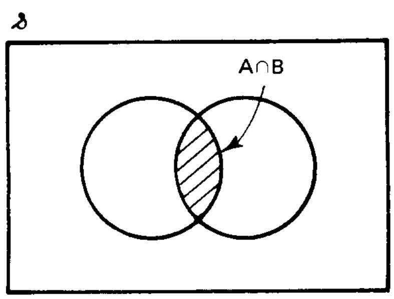
This is called the "or" rule of probability.

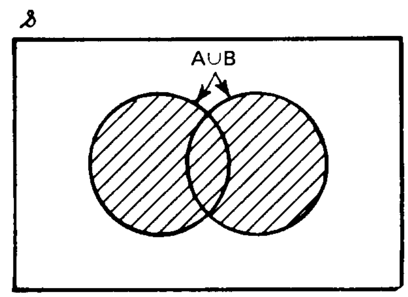
Probability Rule 5. When one trial has no influence on another, their joint probability is equal to: $$p(A \cap B) = p(A)p(B)$$

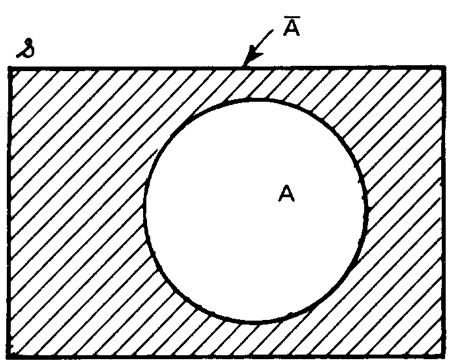
This is called the "and" rule of probability.

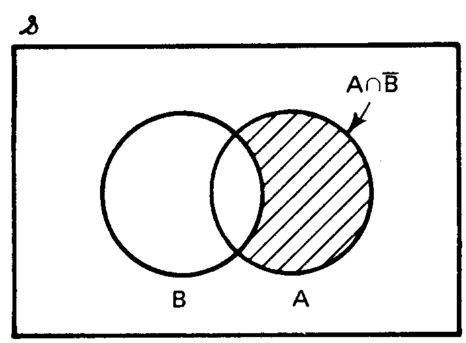

S
A

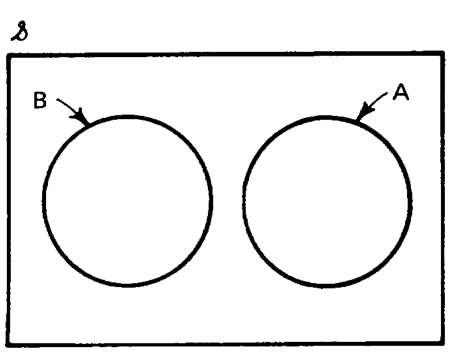

S
B
A

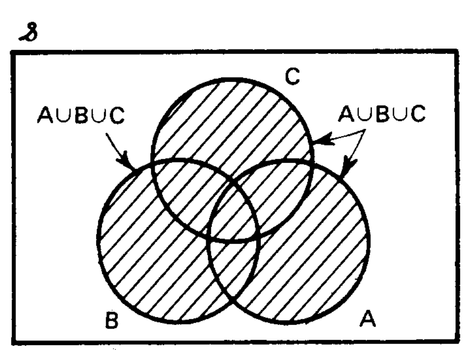

S
A∩B


S
A∪B


Ā
S
A


S
A∩B̄
B
A


S
B
A


S
C
A∪B∪C
A∪B∪C
B
A