Some Non-Linear Dynamics of Group Dynamics

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Abstract

I present a summary of progress in a research program begun by Leik and Meeker (1995) which uses models derived from the well-known Lotka-Volterra (1931, 1932) model of species competition. Leik and Meeker adapted Lotka and Volterra's model to apply to social interaction by adding assumptions about equity concerns of interacting members of groups. This research program uses computer simulation methods to predict outcomes of different conditions and to estimate parameters from actual data. I first describe the logic of the models, and then some results from applications to the problem of distribution of interaction in discussion groups. Data from two experiments have provided the opportunity to estimate parameters and test the fit of the models. The main part of this paper is a theoretical extension of the models to other aspects of group behavior. One feature of this set of models is that they can describe the development of either equality or inequality, depending on the starting values of several parameters. I argue that this feature reflects the reality of a number of small group processes, e.g., decision making in communication networks, effects of different levels of task complexity, and effects of legitimacy of status hierarchies. I explore some of the ways in which these aspects of group interaction can be represented by this kind of model. I also discuss some theoretical and practical problems of this research program.

Some Non-Linear Dynamics of Group Dynamics

Introduction

It is well-established in small group research that the nature of a group's task places important constraints on other feature of group process such as equality vs. inequality of participation, leadership style, individual satisfaction, success of group task performance, etc. In fact it is almost a truism that whatever one can say about group processes must be conditional on the logic of the task, and that conclusions are of the type 'under some task conditions A works best or emerges most naturally, wile under other task conditions, the opposite of A works best or emerges most naturally'.

Several examples can be provided. One prominent early case comes from research programs on the structure of communication networks (.e.g., Bavelas, 1948; Glanzer and Glaser, 1961; Shaw, 1964; 1978). This program began with the question: is a more centralized communication structure more efficient than a decentralized (egalitarian) structure? Early answers seemed to that it is; however theses studies were all conducted with groups working on simple tasks primarily requiring coordination of information exchange. When similar groups were given complex tasks requiring extensive information processing suddenly less centralized structures were more efficient. Shaw (1978) has interpreted these differences as due to the effects of position independence (the degree to which a group member can work independently of others) and saturation (the degree to which task demands become more than one person can handle). Complex tasks quickly cause a central position to become saturated, making the group less efficient than one in which there is more equality of participation.

Another example is from a research program on leadership style and group effectiveness conducted by Fiedler (e.g., 1977). His conclusions were that under conditions of a task that requires primarily coordination a task-oriented, highly directive leadership style produces better group performance whereas with a task requiring a lot of individual effort of ambiguous standards a more socially oriented style produces better outcome. Once again, with a simple task requiring coordination having one person make all the decisions saves time but with a complex task time is wasted because one person can't do it all.

In general, the degree to which one person's actions depend on another's, and the degree to which task efforts get in the way (as with saturation) are important intervening factors in the complex relationship between group structure and task performance. (See McGrath, 1984, for a discussion of different dimensions of tasks and their relationship to other group processes.)

In this paper, I explore a set of mathematical models that deal with the effects of interdependence, and show how they can help elucidate some of the processes. Earlier versions of this work have appeared in Leik and Meeker (1995) and Meeker and Leik (1997).

Types of task interdependence

Consider a group of two actors, a dyad or a larger group taken one dyad at a time, or one leader and the 'rest of the group'. If these actors each contribute to a group product, there are a variety of ways in which their actions can either enhance or inhibit each other's performance. Our model can describe some of these.

The basic model for task interdependence is expressed by the following two equations (from the species competition, or Lotka-Volterra model adapted to describe the level of task output of two individuals in a task oriented group); Lotka (1932), Volterra (1931); see also Felmlee, and Greenberg. (1999), Huckfeldt et al.(1982).

$$O_{1,t} = O_{1,t-1} + O_{1,t-1}(- R_1 * O_{2,t-1} - S_1 * O_{1,t-1} + C_1) \quad (1.1)$$
$$O_{2,t} = O_{2,t-1} + O_{2,t-1}(- R_2 * O_{1,t-1} - S_2 * O_{2,t-1} + C_2) \quad (1.2)$$

For example, a task oriented discussion group has two 'competitions'; one of time (only one person can talk at once) and one of influence (only one person's directive act can prevail at once). Some tasks impose additional sources of competition; for example, when a task requires a lot of information processing, individuals can become 'saturated' and slow down or begin to ignore some information. In general, research on groups that work on tasks involving coordination and/or information processing has routinely shown the structure of the task to have a profound effect on group interaction and outcome.

In words, the model says that amount of output for each actor at each time point is the sum of the previous output plus (or minus) an increment based on the previous output weighted by the actor's Reactivity to the other's output, the actor's Self-response to own previous output, and a constant characteristic of each actor.

To understand the dynamics of these processes, let us consider what happens with only one actor. If only one species (or one person working alone at a task) produces output, then there is no parameter 'R' because there is no Other to react to. If there is no Self limitation, then the parameter 'C' governs the Output and it increases without limit (see figure 1).

-Figure 1 here -

However, there may be limitations on each actor's output due to his or her own past output. These sources include: Satiation (boredom), meaning that the added motivation for each additional act is less the more the actor has performed this action in the recent past; exhaustion of resources, including energy, money, and time so that each additional act has fewer resources to support it; and saturation, meaning that the task itself becomes more complex as acts accumulate, for example, with information processing the difficulty of dealing with each additional item of information increases as more items of information come in. Another example of saturation might be filing items in a cabinet with limited space; the more items already filed, the harder it is to find a place for the next one.

If there is a Self limitation, as for example through fatigue, satiation, using up of one's own resources, or 'saturation' as the task itself becomes more complex with more Output, then the process reaches an asymptote (see figure 2).

-Figure 2 here-

The pattern shown in Figure 2 is a very familiar one; most naturally occurring processes, be they growth of species or amount of individual human behavior, are self-limiting, reaching an asymptote or 'equilibrium' at which, when other conditions remain stable, the rate of increase levels off.

Next, let us consider two actors working together on a task (or two species existing in the same ecological niche). If a task is independent and 'additive', the contribution of each group member occurs independently of the amount contributed by others, and is added into a group output. In this case, the parameter 'R' will be zero, since there is no interference between actors. If there is also no self-interference, the parameter 'S' will also be zero, and the process will be governed entirely by the parameter 'C' the two actors' constant predilections to perform. Figure 3 shows the result of making this assumption.

The two actors in Figure 3 are escalating without bound (the simulation stops at Output equal 10,000), just as the single actor did in Figure 1. Now, to parallel Figure 2, consider two actors with an independent, additive task and some self-limitation. Here, the parameter 'S' will be non-zero

-Figure 3 here-

but the parameter 'R' which represents task interference between the two actors will be zero. Figure 4 shows the result.

-Figure 4 here-

This type of task structure (independent, additive tasks with some self-limitation) supposes that the parameter 'S' is non-zero. We also include the parameter 'C' representing sources of task output not affected by own or other's past performance. Figures 3 and 4 shows a rather low level of Self limitation (.05) and, as should be expected, figure 4 shows both actors behaving exactly the way a single actor, leveling off at 4.00 acts per time unit. Since with group task performance we are interested in total group performance as well as in the contributions of individuals, the 'Total' for the dyad, i.e. the sum of Output of the two actors is also shown on the graph.

For comparison, Figure 5 shows the same assumption (only self-interference and Continuity) but with a higher value for 'S', .70.

-Figure 5 here-

In figure 5 we see that both actors have leveled off at a much lower rate of Output (.268 per time unit) as we might expect if their tasks impose greater satiation or saturation.

Finally, consider the effects of mutual interference, as manifested by adding the parameter 'R' to the model. Reactivity means the amount #1's Output decreases with the square of each unit of #2's Output. Tasks may exhibit mutual interference through outright competition (only one person can be elected chair), through scarce resources (if one person is on the phone the others can't use it), through the necessity for coordination (if one person talks, others must listen) or interpersonal saturation (if many people send messages to a central position, the person in that position cannot process them all at once). In cooperative task groups there also may be a type of task in which only one person is required to perform the task (if one person turns on the light, the room is illuminated for everyone). This is another type of 'mutual interference' in that if one person does the task, others don't need to. (This would be a 'unitary, disjunctive' task in the classic typology of tasks developed by Steiner, 1972). The more Output #2 has, the more #1 perceives his/her output is not necessary.

Adding mutual interference leads us to the Lotka-Volterra species competition model, in which it is assumed there is both self-limitation and mutual interference. Figure 6 shows the results of assuming that all three processes, individual predilection for action (parameter 'C'); self limitation (parameter 'S') and mutual interference (parameter 'R') operate.

-Figure 6 here-

The relative values of the parameters 'S' and 'R' in Figure 6 are in the range that Lotka and Volterra pointed out will lead to the extinction of one species, i.e. $S_1 * S_2 < R_1 * R_2$. In the concepts of task interdependence, the amount of mutual interference is greater than the amount of self-limitation. Note that the outcome is that one actor (Actor 1 in this case) is doing all the work and the other is doing none, also that the amount of work being done by Actor 1 is the same as under the assumption that Actor 1 works alone but with the same degree of self limitation. This means that the total amount of work (4 units of Output for each time unit) is the same as if Actor 1 started out alone.

Since the two actors in figure 6 began with identical characteristics, parameter values and initial Output, the differentiation is due to which acted first. Although in some respects an artifact of the way the simulation is set up, this is compatible with results of some group process research; often it is the first person to act who takes over the group. We see here how this may be a result of the type of task.

Now, consider what will occur if the amount of mutual interference remains the same but the amount of self-limitation increases (once again, identically for both actors). For example, the individuals' jobs become more complex or they become fatigued at a faster rate while difficulties of coordination remain the same (perhaps the organization for which the actors work has downsized so that each has more additional responsibilities). Figure 7 shows this.

-Figure 7 here-

Notice that now, the outcome is equality; both actors produce at the same rate (.8 acts per time unit, for a group total of 1.6 acts per time unit). The structure has imposed equality rather than differentiation.

For comparison with an independent, additive task with the same level of self-limitation, consider figure 8.

-Figure 8 here-

Without the inhibiting effect of mutual interference (the parameter 'R') the level of Output of the two actors rises to 1.33 each, for a total group Output of 2.66 per time unit.

Now, to see just how delicate the system is as regards the relative values of the products of the parameters 'R' and S', compare figures 9 and 10. These show what happens as the value of 'S', self-limitation approaches and then dips below the range that produces equality. The change is from to .11 (just above), fig 9, to .09 (just below), fig 10 with 'R', mutual interference, still at .10.

-Figures 9 and 10 here-

Figure 10 shows a very different social structure from Figure 9; in Figure 10 one actor winds up doing all the work and the other actor none whereas in Figure 9 the two perform equally. In other words, in figure 10 there develops a division of labor, while figure 9 has none. The total amount of work is similar; 2.22 acts per time unit in figure 9 and about 1.9 in figure 10. Another interesting point is that the process takes a long time to work out; the 200 time points we used in the previous graphs are not sufficient. Showing 1000 time points shows us that the process in figure 10 stabilizes at about 850 time points. The change in job description or work habits that describe the difference between figures 9 and 10 are small and take a long time to manifest themselves; however, they are dramatic as regards the equality or differentiation that result. Since the outcome of differentiation or equality results from the relative values of the 'R' and 'S' parameters, a similar outcome will occur if Self limitation remains constant and mutual interference ('R') changes; a decrease in 'R' to below the tipping point will change a differentiated system to an equal one, and likewise an increase in 'R' may change an equal system to a differentiated one.

We may also wonder about the effects of absolute values of 'R' and 'S'; if the relative values begin in the range that produces equality, for example, and remain in the range but both increase, what happens? One example is shown in figure 11, in which the values of 'S' and 'R' are higher (.9 and .8 respectively) but in approximately the same proportion as in figure 7.

-Figure 11 here-

Here we see that the primary effect of increasing the difficulty of coordination (mutual interference) and self limitation at the same time is to decrease the level of Output; now both actors are producing about .22 acts per time unit, for a group total of .44, at equilibrium. It is intuitively plausible that as both difficulties of the task and of coordination increase, task output decreases.

-Figure 12 here-

For comparison, figure 12 shows a parallel picture to compare with figure 6, i.e.

parameters with relative values producing differentiation but higher values of both parameters, .9 and .8 for 'R' and 'S' respectively.

Figure 12 shows that now Actor 2 is carrying the whole burden, with Output of about .25 acts per unit of time. (There appears to be only one line on the graph because since Actor 1's Output is zero, the Total is the same as Actor 2's Output, and Actor 1's Output coincides with the bottom line of the graph).

Other Sources of Inequality

Thus far, we have considered only the case where the two actors begin with identical situations; same values of all the parameters, same initial Output, etc. What will happen if the two are not identical? Figure 13 shows one such condition. Here, the relative values of 'R' and 'S' are in the range producing equality but they are not equal; the value for both self limitation and mutual interference for one actor is twice what it is for the other (perhaps one is new and has more difficulty with the job, or they have different tasks).

-Figure 13 here-

This produces an unequal outcome, with Actor 2 (who had lower levels of both mutual interference and self limitation, i.e. had an easier time with the task) producing more than Actor 1 <u>but</u> with Actor 1's Output still not reaching zero as it did with the parameters in the range producing differentiation. At about time 41, Actor 1 is producing .048 acts per time unit and Actor 2 1.976, for a group total of 2.024 per time unit.

This might represent a task in which both actors continue to contribute but one contributes more than the other does. It is interesting to note that the total group Output is highest early in the process, while the two actors' Outputs are differentiating. The differentiation is smooth, and begins with the first round of interaction.

If the parameters are not identical but begin with relative values in the range that produces all-or-nothing differentiation, something like the example in figure 14 may occur.

-Figure 14 here-

This shows a rapid all-or-nothing differentiation, with Actor 1 producing the familiar 4 acts per time unit and Actor 2 producing zero.

Another assumption we have so far made is that the two actors begin with identical contributions on the first round of interaction (we have assumed each starts with a contribution of 1 act). Examples of what may happen if we vary this to assume that one actor makes a larger contribution than the other does, but the parameters are the same are shown in figures 15 (parameters in equality range) and 16 (parameters in all-or-nothing differentiation range).

-Figures 15 and 16 here-

In both of these examples, the process eventually stabilizes at exactly the same outcome produced by these parameters with identical initial contributions. (Compare figure 15 with figure 7 and figure 16 with figure 6)

Finally, we can consider the role of the parameter 'C' which has no relationship either to own or to other's previous output. We have characterized it as representing the individual differences, role behaviors, or other features of the setting that are not directly affected by level of output. What happens if 'C' is zero, i.e. there are no such effects? Or, what happens if 'C' is much higher than the value of .20 we picked for our other examples? Figures 17, 18, 19, and 20 explore these questions. Figure 17 shows parameters in the range for equality and with a high value of 'C', figure 18 shows the same 'equal' range but with 'C' set to zero. Figure 19 shows parameters in the range for all--or nothing differentiation and a high value for 'C' and figure 20 shows all-or-nothing parameters and 'C' set to zero.

-Figures 17, 18, 19 and 20 here-

These figures indicate that the effects of 'C' are on the speed with which the process develops and the ultimate value of Output reached. However, 'C' does not create the equality or differentiation that the other parameters do when the values for the two actors are equal. However, when 'C' is not the same for the two actors, their outputs differentiate, as shown in figure 21. –

Figure 21 here-

This comes close to the more usual interpretation of differentiation within groups, an explanation based on pre-existing individual differences. When 'C' is higher, the actor may be seen as trying harder, having higher status value, being better trained, etc.

Supposing parameters can change during the interaction

The models presented thus far assume that parameters are fixed throughout the interaction. In the next phase of this work, we consider some models that suppose that the parameters may change in systematic ways during an interaction. This will, of course, affect the outcomes, both level of individual and group task output and whether the outcome is equality of contribution, differentiated contributions, or all-or-nothing differentiation.

Consider the effects of changes in parameter "R". There may be two possibilities:

a) the task has no flexibility; there can be only one chair, or the task genuinely requires only one person to do it. Here, Reactivity is Fixed.

b) the task appears at first to be competitive or one person only, but in fact group performance can be improved by everyone's contributing. The results of acting as if a) holds are performance losses due to social loafing or to inappropriate status generalization. How might this be avoided? One way is for group members to monitor their relative levels of contribution, with the idea that equal contributions are the fairest. If contributions begin to depart from equality, some change in rate of responding to other's Output may occur. If norms of fairness are implemented, Reactivity may be modified.

Now, under what combination of other assumptions will

1) output stabilize

2) total group output be high

3) both actors contribute (that is, outcomes, if differentiated are not all-or-nothing differentiation)?

Assume that the other parameters are Fixed (Self Response is the amount #1's Output is inhibited by own past Output, causing a leveling off of Output over time; Continuity is a stable 'personality trait').

Assume that both actors begin identically; same initial output, same values of parameters, same equations for changes in Reactivity. Also, since we are working with the assumption that the actors value equality of contribution, assume that the initial values of the Reactivity and Self Response parameters are in the range that will produce equality.

Next, we change the assumption that Reactivity is Fixed, and assume that both actors will monitor the relative levels of output and adjust their rate of Reaction to other's past output according to the protocol UP (that is, the 'interference' of other's output increases as Actor gets ahead in relative Output). This might be interpreted as 'allowing the other to do a larger share'). As Actor realizes he/she is contributing relatively more than Other, Actor responds to Other's Output by decreasing own at an increasing rate).

We need to recall that if this change in parameters puts the relative values of Reactivity and Self-Response into the range in which the product of the two reactivities is greater than the product of the two Self-Responses (which remain Fixed) then the task is no longer 'equal outcome' but will lead to differentiation instead.

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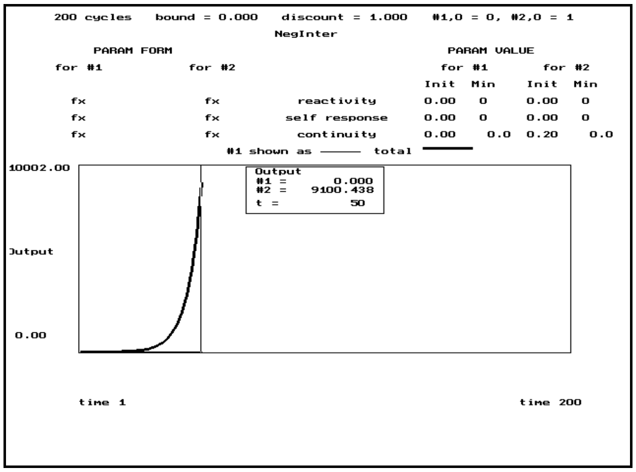


Figure 1 One actor, no limitation

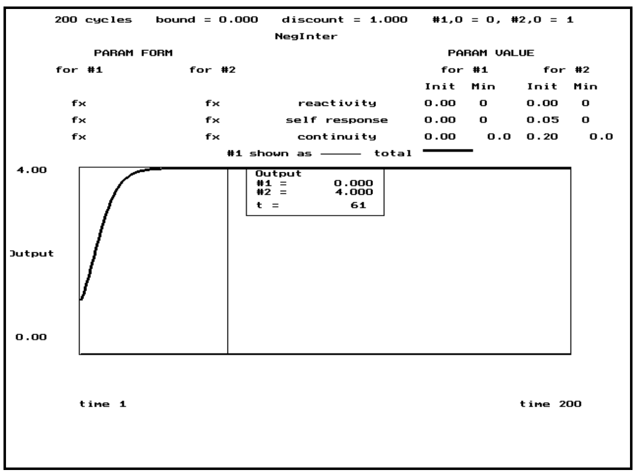


Figure 2. One actor, Self limitation

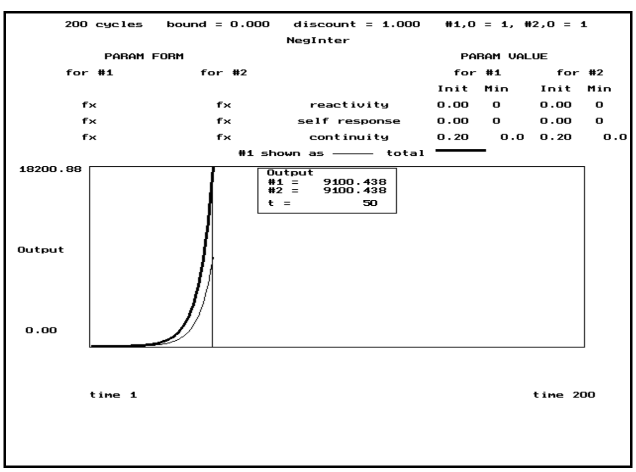


Figure 3. Two actors, additive task, no self limitations

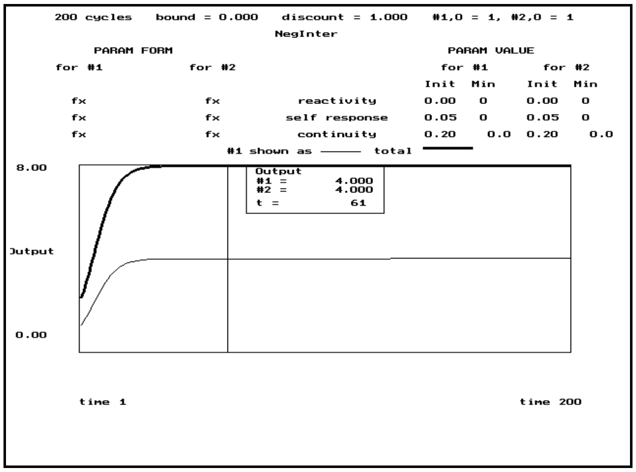


Figure 4. Two actors with no mutual interference but self limitation

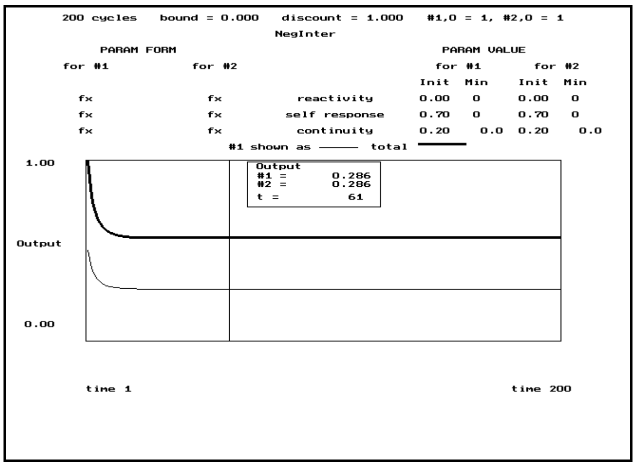
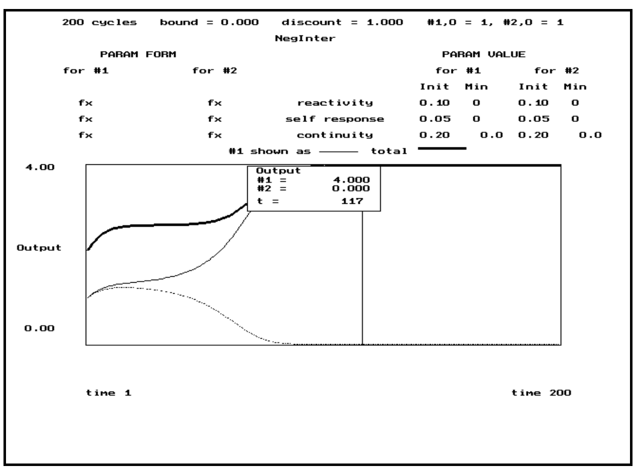


Figure 5. Two actors, no mutual interference, higher self limitation

Figure 6. Two actors, both self-limitation and mutual interference. 'unequal' outcomes, low levels of parameters



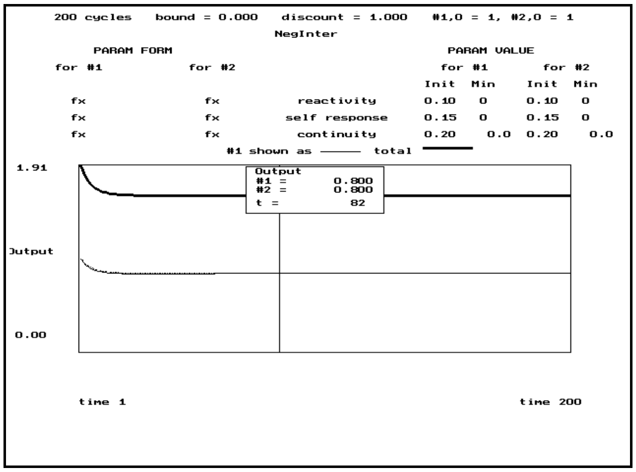


Figure 7. Two actors whose job difficulty has increased

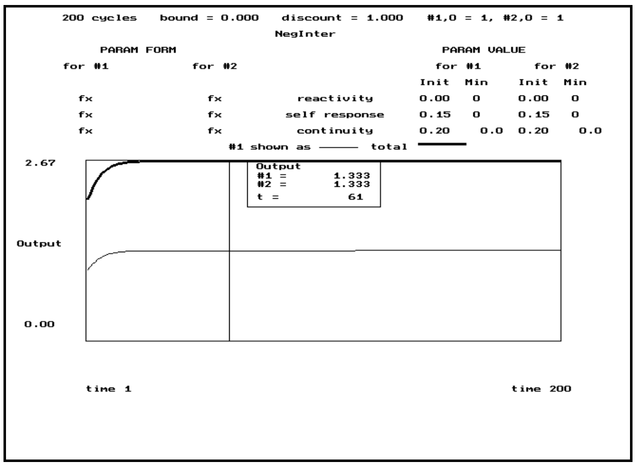


Figure 8. Two actors, no mutual interference, self limitation of .15 (compare fig 7)

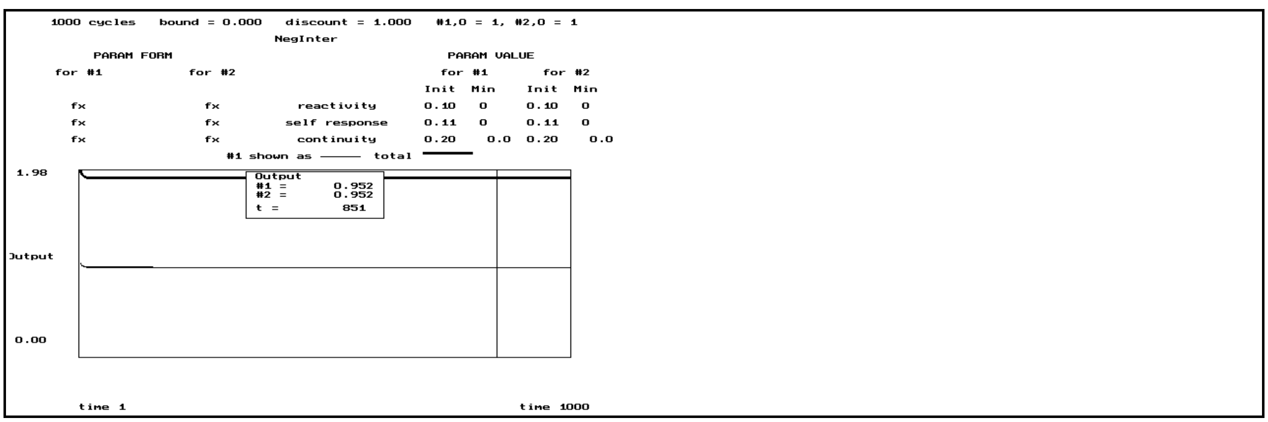


Figure 9. self limitation just above values producing equality

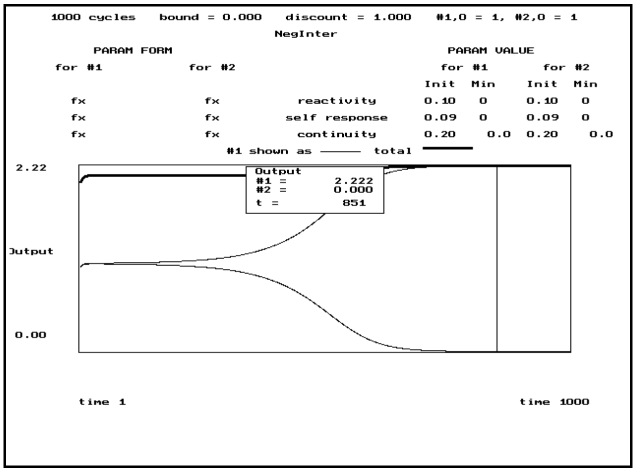


Figure 10. Self limitation just below range producing equality

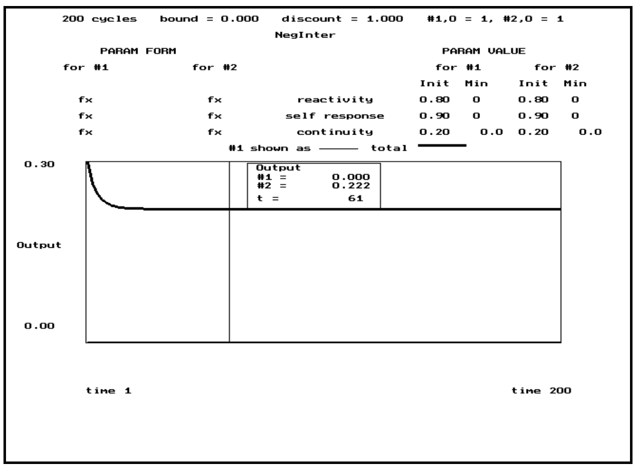


Figure 11. Equality, with higher levels of mutual interference and self limitation

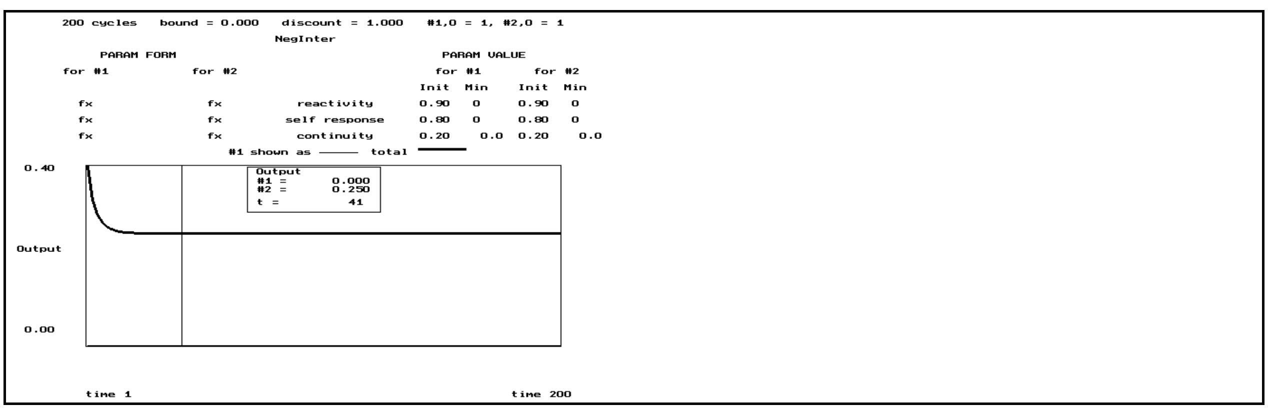


Figure 12. Unequal outcomes, higher levels of mutual interference and self limitation

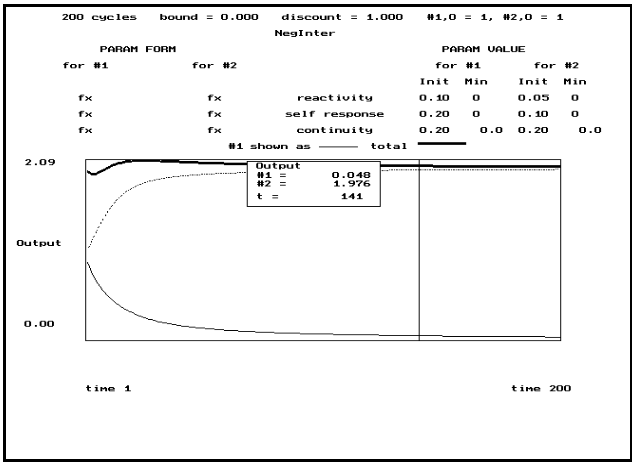


Figure 13. Two different actors

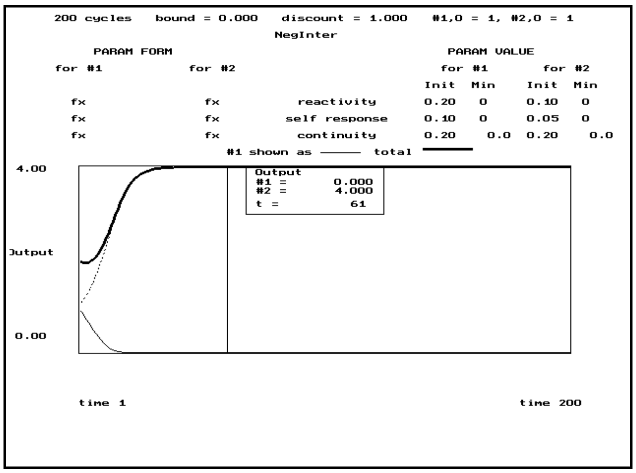


Figure 14. Unequal parameters, all-or-nothing differentiation

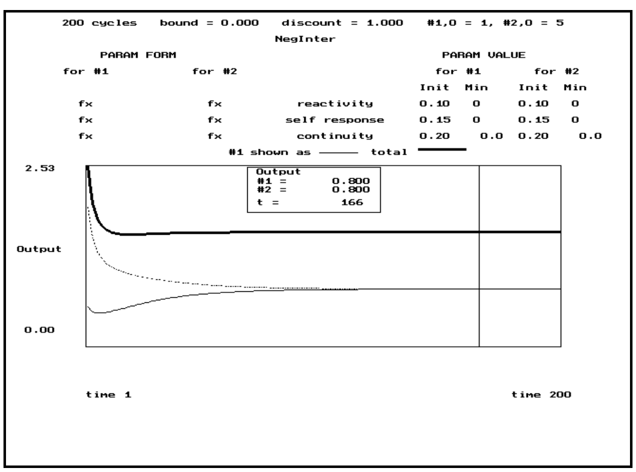


Figure 15. Unequal starting contributions, equal outcomes

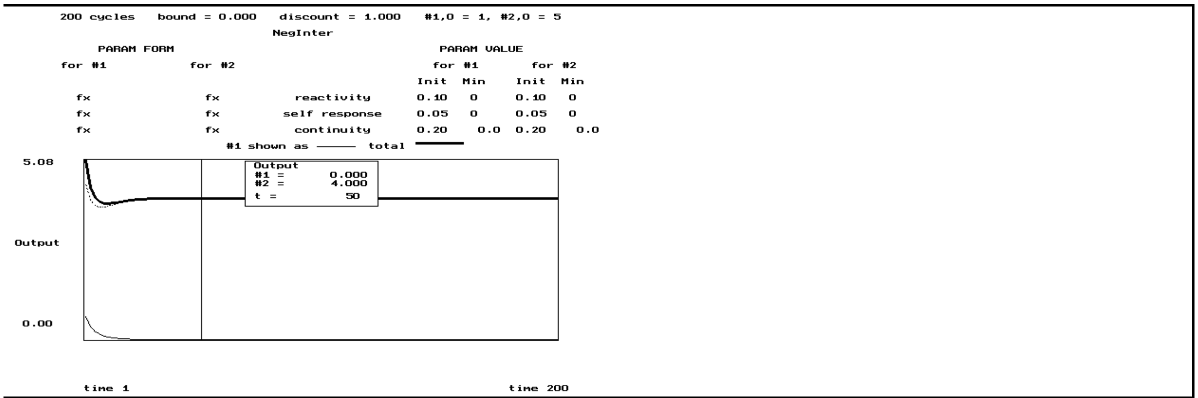


Figure 16. Unequal starting contributions, all-or-nothing differentiation

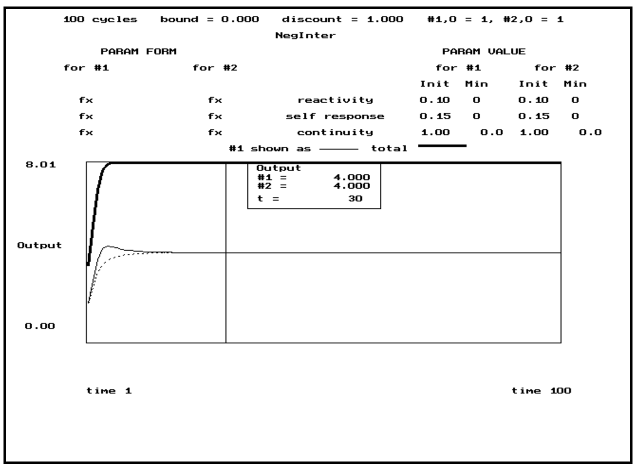


Figure 17. C is high, equality

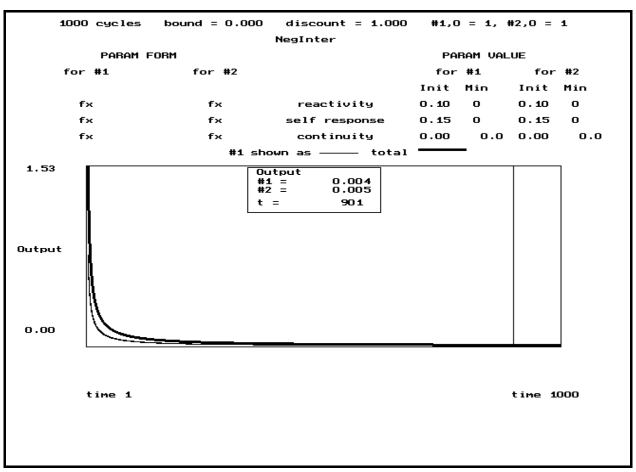


Figure 18. C is zero, equality

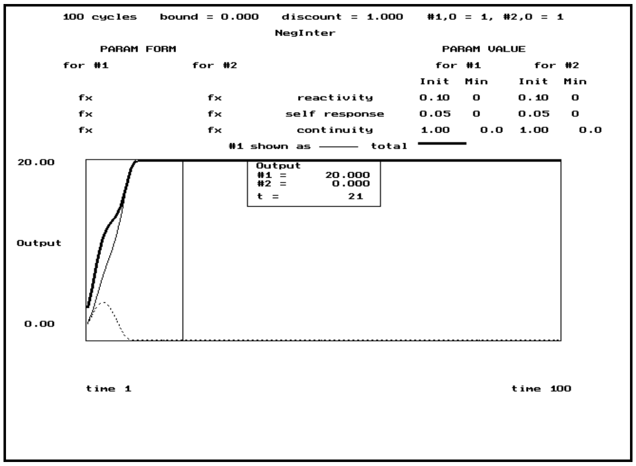


Figure 19 C is high, all-or-nothing differentiation

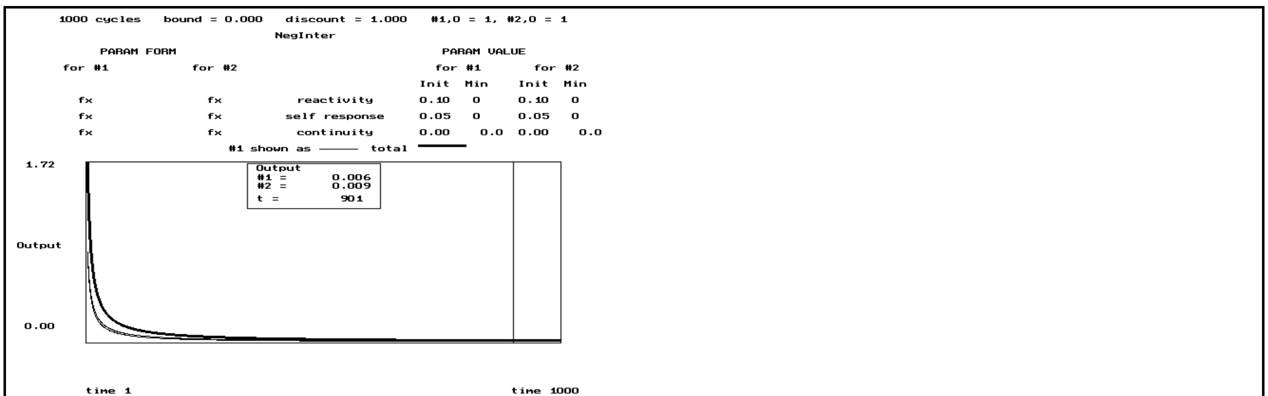


Figure 20 C is zero, all-or-nothing

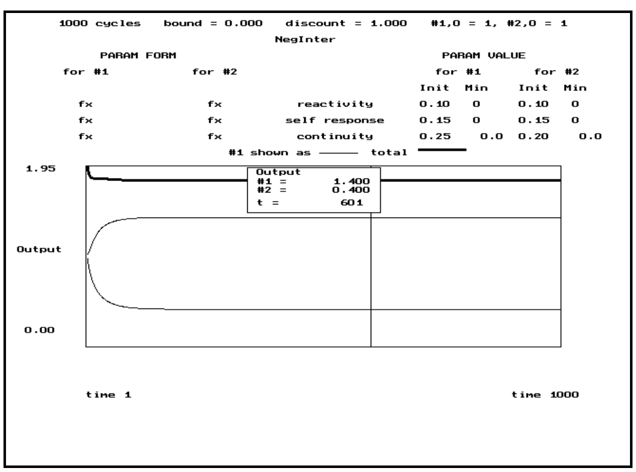


Figure 21. Different values for 'C', otherwise 'equal' parameters